



TITLE:

# THE LOGIC OF UNCERTAINTY ACCORDING TO J. M. KEYNES

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CITATION:

Hishiyama, Izumi. THE LOGIC OF UNCERTAINTY ACCORDING TO J. M. KEYNES. Kyoto University Economic Review 1969, 39(1): 22-44

ISSUE DATE:

1969-04

URL:

<https://doi.org/10.11179/ker1926.39.22>

RIGHT:

# THE KYOTO UNIVERSITY ECONOMIC REVIEW

MEMOIRS OF THE FACULTY OF ECONOMICS  
IN THE KYOTO UNIVERSITY

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Vol. XXXIX, No. 1

APRIL 1969

Whole No. 86

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PUBLISHED BY

THE FACULTY OF ECONOMICS, KYOTO UNIVERSITY

SAKYO-KU, KYOTO, JAPAN

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# THE LOGIC OF UNCERTAINTY ACCORDING<sup>1)</sup> TO J. M. KEYNES

By Izumi HISHIYAMA\*

## I

I think that one chapter has been left untouched in the studies about J. M. Keynes which have been made so far. It is none other than the relationships between *A Treatise on Probability*<sup>2)</sup> and *The General Theory*. Because *A Treatise on Probability* was Keynes' first painstaking work<sup>3)</sup>, I believe in a certain sense that the study of these relationships will serve to clarify the changes of his thought from his early days up to the latter.

Though it is to be expected that some would say that setting up a proposition like that would be quite meaningless in the case of Keynes, I think that even if a doubt of that kind might be aroused, it would be natural in the case of Keynes. It seems to me that among all the prominent scholars in the past and present there are few scholars like him who have been unsophisticated out of a bad habit of insisting on the conventional way of thinking. Of course he has never insisted on his own assertions. It is likely that some may dare criticise him, saying that he is eccentric. As is well known, in his *Treatise on Money*, 1930, he criticised the formula (the cash balance equation) asserted in his *Monetary Reform*, 1923, and also in his *General Theory of Employment, Interest and Money*, 1936, he abandoned the essential part (fundamental equations) of his *Treatise on Money*. He further proceeded to make a severe, if not fatal, criticism of the classical economists, among whom he himself belonged for a long time and he attempted to extricate himself from their field. Viewed from this angle, I would rather think that a deeper develop-

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1) This paper has the two succeeding ones: "Homo Oeconomicus" in J. M. Keynes' *Theory of Investment*, *Shiso*, No. 519, Sept. 1967; and "J. M. Keynes' Logic of Uncertainty and Money", *Shiso*, No. 526, Apr. 1968.

2) J. M. Keynes, *A Treatise on Probability*, 1921. This book has been reprinted several times since 1929. The page numbers which are given to the lines quoted are taken from the reprinted edition, 1962, published by Harper & Row.

3) This book is Keynes' first painstaking work which originally was prepared as a fellowship dissertation (1908) of King's College (Cambridge).

ment of one's thought would, above all other things, have meant to Keynes to criticise the very concept in one's own mind, to make a conquest of it and to attempt to extricate oneself from it.

On the other hand, it is also commonly known that he had something of the personality of a current critic having an acute realistic sense. Thus no point of the vicissitudes of current problems ever escaped being timely grasped by Keynes' sharp sensibility. The innumerable variety of economic problems which had given rise with bewildering rapidity during the period from the end of the World War I to the great depression, such as the problems concerned with post-war disposal and rehabilitation measures, the system of currency and price stabilisation, chronic unemployment and depression and what not, together with their resulting economic diseases, came into existence, carrying particular phases and colours in each case. As it acted in concert with each case, Keynes' diagnosis and his prescription slips showed some changes one after the other. In this way he never stayed at the same place; just as if he were running water. Whatever specific time he may be taken up at, we always find him in the posture of a man of thought who is attempting to face the future by doing away with the past. In a word, he is, as it were, always "a man on his way". If one turns his mind toward the past from the viewpoint of such man of thought, the past must have reflected on his mind not as what should be taken over but as what should be abandoned. The past must be something incomplete or immature, if not something erroneous. The harder the effort is made to trace the changed course of his original thought, the oftener the chances to think in that way increase. If viewed from this aspect, it may appear to be meaningless to make a quest for the relationships between his *General Theory*, which was intimately conceived by Keynes, and his *Treatise on Probability*, which was the crystallisation of his primitive thought. In this way this very point in question has been almost left (duly ?) untouched by those who have been regarded as specialised Keynes scholars. Neither L. R. Klein, S. E. Harris, A. H. Hansen nor D. Dillard, etc. has made mention of even a few words with respect to the *Treatise on Probability*, not to speak of the above mentioned point in question<sup>4</sup>. Even J. A. Schumpeter, who has been so well-known for his profound knowledge, did not grasp the meaning of the *Treatise on Probability* as having been anything more than "an outlet for the energies of a

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4) L. R. Klein, *The Keynesian Revolution*, 1947; D. Dillard, *The Economics of John Maynard Keynes*, 1948; A. H. Hansen, *A Guide to Keynes*, 1953; S. E. Harris, *John Maynard Keynes, Economist and Policy Maker*, 1955.

mind<sup>5)</sup>. Because many other scholars one after another have done nothing but contribute to the spread of such views, the specific problems concerned with a *Treatise on Probability* have been left untouched, being buried in oblivion for the period of thirty years since the publication of *The General Theory*, and there has been at least no room left in any economist's concern to be taken up by these problems.

It may be said that the reason why a *Treatise on Probability* has been disregarded by those who have made a study of Keynes is, not because it was immature or that it has been replaced by his own thought in later days, but because it was an achievement in a field having nothing to do with economics, or because it had no concern whatsoever with the formation of his economic thought. Maybe it was assumed that the line of thought in terms of a *Treatise on Probability* and that which has penetrated through *The General Theory* by integrating the thoughts in the field of his economics were, so to speak, two parallel lines, neither of which would ever cross the other. Or rather, the truth might lie in the fact that a *Treatise on Probability* has never been read, and that those specialists interested in this field who perused the theory so happened to show no interest in Keynes' economics. Let it be what it may, it is anyway an indisputable fact that the re-examination of the part that the *Treatise on Probability* played in forming Keynes' thought, specifically in systematising his economic thought has never been attempted before, being completely neglected by those who have made a study of Keynes' works. And if it can be assumed that such a point in question does have some weight which can not be disregarded, then it follows that the various studies conducted about Keynes so far are characterised by a blind point of a serious nature. As for me, I am of the opinion that this point in question should by no means be neglected, and therefore my judgement leads to the conclusion that the studies about Keynes in the past are not free from such a blind point.

The *Treatise on Probability* is not a book that deals with the technique and mathematical application of probability as the title may suggest. It is a product of a far more extensive and ambitious way of grasping the nature and role of probability. Speaking briefly, its aim is to compose a new logical foundation rather than the creation of a new logic on the basis of empirical science. Such a way of taking up the problem is very clearly expressed in his quoted line of Leipzig's words, which may be cited here as follows: "J'ai dit plus d'une fois qu'il faudrait une nouvelle espèce de logique, qui traiterait des degrés de Probabilité." (p. 3.) Since the main subject of a *Treatise on Probability* lies in the clari-

5) J. A. Schumpeter, *Ten Great Economists, From Marx to Keynes*, 1952, p. 269.

fication of 'a new type of logic' (une nouvelle espèce de logique) to provide a logical basis of empirical science, it would not be entirely wrong to say that this book treats the methodology of empirical science, if expressed in recent words. Therefore, the relationship between a *Treatise on Probability* and *The General Theory* can be, in a sense, comparable to that between J. S. Mill's *System of Logic* and his *Principles of Political Economy*, or that between Jevons' *Principles of Science* and his *Principles of Political Economy*.

For the purpose of making a thorough study of the meanings of J. S. Mill's thought on economics it would probably be essential to make a close examination of his system of logic. The same should be held true with regard to the case of Keynes. That is, for the purpose of clarifying the theoretical meaning of his thought on economics developed in his *General Theory* it would be necessary to examine his system of logic. In the meanwhile, speaking of the system of logic of Keynes, there is no such book except his *Treatise on Probability*, and it can be seen that Keynes sacrificed his whole youthful life, when his intellectual energies were brought into full play, only to complete this book in which he wanted to accomplish the task of creating a new type of logic which could well be set up against Mill's system of logic<sup>67)</sup>. This book is not anything like a layman's piece-work, but is literally one of his life works<sup>8)</sup>, as commented on by R. F. Harrod. It appears to me, as far as my view is concerned, that Keynes' philosophical thought, being different from the case of economic thought, grew to maturity in his youthful days and that his systematical thought of logic which bloomed in his youth never came to be revised in all his life. In the case of Keynes, if described in a more or less exaggerated manner, his philosophical vision with respect to the methodological foundation of his theoretical framework came to be established during the period of his earliest intellectual formation, and the development of his thought filled with seeming diversities and changes that took place afterwards, up to such time when his *General Theory* was published, was something like sparks

6) Keynes declared in the introduction to *A Treatise on Probability* that this book was on the track of 'the English tradition', such as Locke, Berkeley, Hume, Mill, Sidgwick, etc., but that on the other hand Mill's *System of Logic* showed a "complete failure to grasp with any kind of thoroughness the nature and the importance of the theory of probability" (p. 268, note 1).

7) Keynes devoted all his leisure hours to the study of *A Treatise on Probability* during the period covering 1906-1911, that is, from 23-29 years of age. Cf. R. F. Harrod, *The Life of John Maynard Keynes*, 1951, p. 133.

8) With respect to *A Treatise on Probability*, Harrod made the following statement: "His treatise must be regarded as embodying a substantial proportion of his best life-work. It would be quite wrong to think of it as *jeu d'esprit* thrown off by an economist to show that he had some philosophical capacity also." *Ibid.*, p. 133.

flying apart just before a gigantic lense set in the frame of such a vision was about to be focussed on a certain point. It appears to me that those who made a study of Keynes in the past have all been dazzled too much by such brilliant sparks and that they have failed to recognise the other side of Keynes' thought—that element which kept wandering in hope of finding a place to settle down, taking root deep in the basic layer of his superficially ever-changing thoughts, i.e. that ever-lasting philosophical element.

Be the matter as it may, how suggestive it is that the very first field of study from which Keynes set forth to begin his scholastic itinerary was principally concerned with the theory of probability and the theory of money. To recompose the theory of probability as a fundamental logic common to the empirical sciences by straightforwardly accepting the uncertainty which spreads its wings in the field of experience as a natural and normal element on the one hand, and to make complete research of a theory and practice of all monetary phenomena in which uncertainty makes its most prominent appearance in the economic field on the other—all these were none other than the subjects of his study to be pursued by himself when Keynes launched himself in to the study of a science. As already mentioned, it seems that the study of the systematisation of empirical and inductive logic based on the concept of probability was completely finished in his early days, being maintained as such without any modification throughout all his life. Contrariwisely, however, the systematisation in the field of his economic thought, which was started from his study of money and which was integrated in a voluminous *General Theory*, through a series of various works on money and banking, was characterised by repeated destruction and creation, criticism and reconstruction, being forced to pass through a long painful gestation period. Disregarding the danger of too much simplification, if the formation of Keynes' thought is to be briefly and symbolically summarised, it can be reduced to the following two fields of systematisation: (1) *logic* based on *probability* and (2) *economics* based on *money*. Now then, our present problem is to examine whether or not an everlasting straight line originating from *probability*, and a diversified, and so to speak, spiral line originating from *money* respectively, determine each independent field of its own which will never cross the other. Needless to say, I think that these two fields are connected by close relationships with each other, and that in the intersection of these two lines should lie the *raison d'être* of a new microcosm, in opposition to the old classical microcosm which is composed of Mill's inductive logic and deterministic economics mainly concerned with *real* matters taken over by A. C. Pigou through

the ideas held by J. S. Mill and A. Marshall.

Viewed from this angle, it follows that the system of *A Treatise on Probability* should be regarded as an essential component element of the thought of Keynes, being coupled with the system involved in *The General Theory*. Therefore, it can fairly be said that almost all of the scholastic studies and literature in the past<sup>9)</sup> which did not take this point up deviated from the fundamentally important point in question, if viewed both from the standpoint of a far-sighted grasping of the systematical thought of Keynes and from the standpoint of the clarification of the theoretical meaning of his economic thought. Consequently, the subject matter of my present study about Keynes, in brief, can be focussed on clarifying that the two above-mentioned different categories of thought, or fields, are connected together in meaningful relationships, that the two lines originating from *probability* and *money* respectively should intersect, and still further that his systematised economic thought should be rebuilt on Keynes' logic of uncertainty.

## II

It seems that among Keynes' systematised economic thought concentrated in the *General Theory*, one of the groups of problems which has some relationship with the thoughts concerned with the theory of probability is *the theory of inducement to invest or the theory of investment decisions*, and another one of them is *the theory of propensity to consume or the theory of the consumption function*. Now, as is generally known, since these two problems are no more than the essential elements of the so-called principle of effective demand, if it could be made clear that these theories are based on a consistent ground in terms of the theory of probability, then our assumption that the Keynesian theory of probability has some meaningful relationships with his systematised economic thought could be corroborated. However, even if it is made clear that the theory of probability is related to the theory of investment inducements and the theory of the consumption function, it does not follow that the way in which the relation comes to arise and its nature are exactly identical in both cases. In my opinion the theory of probability and the theory of investment inducement are directly related, and such a combination is very natural.

9) As far as my knowledge is concerned, some examples of exceptions in which this point in question have not been disregarded are seen in the following two works: Yuzo Yamada, "Introduction 'About the Keynes' "Treatise on Probability", in Keynes Research Association (ed.), *Studies of Economics of J. M. Keynes (Keynes Keizaigaku Kenkyu)*, 1950; G. L. S. Shackle, *The Hedgehog and the Fox*; —, "Keynes and the Nature of Human Affairs" in his work *The Nature of Economic Thought*, 1966.



The reason is because the problem of investment is concerned with the way of selecting a probable course of action under uncertain anticipation. Putting it another way, because the problem resolves itself into a problem of selection or decision under uncertain conditions, the concept of probability comes to be naturally formed in consciousness. Taking the way of thinking advanced in the *General Theory* for example, if a little more concrete explanation is to be given, it runs as follows. Generally speaking, the decision to produce new capital goods, such as machines or buildings, will be governed by the production cost and the expected market price. Now then, when the price of capital goods is expected to exceed production cost, the production of capital goods, i.e. investment will be stimulated. In other words, the scale of investment will be determined at the level where the price and production cost come to be equal. Nevertheless, the price of capital goods depends upon two factors: one is the rate of interest and the other is the expectation of the future profit from capital goods. When the rate of interest goes down, other conditions being equal, the price of capital goods will go up, and similarly when the expectation of the future profit turns out favourably, its price will also go up. The way in which the price of these capital goods fluctuates is a plain fact to be easily understood by analogical inference if our eyes are turned to the daily price fluctuations of valuable securities, although these are not real assets like machines and buildings, which can be looked upon as representative of real capital assets. Now, if roughly expressed, the rate of interest according to Keynes, once the quantity of money is fixed, is determined by the propensity of the public to hold money. However, various motives of the public for holding money in possession for future use are governed by the anticipation or judgement with respect to the future. On the one hand it is needless to say that the anticipation of future profit from capital goods is also to be governed by judgement having an uncertain element. Therefore, it necessarily leads to the conclusion from all that has been stated that, if the production cost of capital goods is given, then investment after all is to be based on two different judgements with respect to the future—the preference of the public to hold money in possession and their opinion of the future profit from capital goods<sup>10</sup>. Because these judgements are of an extremely uncertain nature, they are not based on safe and definite grounds. Putting it another way, in the case of the selection of a course of action, the result of which will become known not instantaneously on the spot but more or less in

10) Cf., J. M. Keynes, "The General Theory of Employment", *Quarterly Journal of Economics*, Vol. 51, No. 2, 1937, p. 218.

the distant future, however complex and confusing its phenomenal form may appear to be, it appears to me that the structure of inference based on the theory of probability has its roots deep in the background of such judgements. Therefore, it can fairly be assumed that the way of thinking in terms of the theory of probability is not inessential in Keynes' theory of investment inducement or the theory of investment decisions, and that there does exist a very close relationship at the very bottom of the foundation.

As to the so-called theory of the consumption function, viewed from the chronological sequence of Keynes' thought, it must have been formed in his mind as a flashing light in relatively later days, possibly immediately after the publication of the *Treatise on Money* in 1930. It can be assumed that the concept of a fairly stable consumption function must have come to his mind through the formulation of an investment-multiplier prepared by his student R. F. Kahn<sup>11)</sup>, as its inevitable assumption, rather than as a result of its by-product. If it is permissible to view it in this way, then it may be considered that the theory of the consumption function, at least as far as its *genetic* ground is concerned, has nothing to do with the theory of probability. If so, where should the relationships between the theory of the consumption function and the theory of probability be sought? As to this question, the following two points seem to be worthy of note: one is the problem of statistical generalisation and statistical stability, the other being the problem concerned with Keynes' way of interpreting the so-called aggregation.

Now, if seen from the standpoint of empiricism, it would be permissible to think that the theory of the consumption function is concerned with a kind of statistical generalisation. If consumption and income are grasped as an aggregated quantity of an economy as a whole, then the theory of the consumption function would indicate the plain fact that the move of aggregate consumption is governed *principally* by the move of aggregate income. Of course, the propensity of the public to consume may be affected by a number of factors, such as conditions of income distribution, their attitudes toward the future, etc., and also by the rate of interest to a certain extent. However, if income increases, consumption increases, too, but the rate of the increase of the latter is smaller than that of the former. In brief, the increment of consumption  $\Delta C$  is always smaller than the increment of income  $\Delta Y$ . Moreover the ratio of  $\Delta C$  to  $\Delta Y$  (this is called the marginal propensity to consume) is positive and less than 1. Keynes asserts that this characteristic prescribes the "normal shape of the function of consumption"

11) R. F. Kahn, "Home Investment and Unemployment", *Economic Journal*, June 1931.

of a society and it forms the "fundamental psychological law" of an economic society<sup>12)</sup>. Now then, if the law pertaining to the function of consumption, to which the characteristic (it may now be expressed by  $\phi$ ) that is mentioned in the above can be ascribed as an inevitable conclusion, is grasped as a kind of statistical generalisation, it is clear enough that it will be formed on the basis of the theory of probability.

A statistical law does not so strictly require universal soundness that even one exception will lead to its abandonment. For instance, although the drawing of a conclusion that *all* swans are white, based on data which show that this or that or those swans are white, may be regarded as inductive reasoning in conformity with the classification<sup>13)</sup> of Keynes' *Treatise on Probability*, it should rather be looked upon not as statistical induction but as universal induction. In a case of statistical induction the proposition to be clarified would show that *most* swans are white, or that the probability that a swan is black is such and such, based on data that this or those swans are white and the other one is black. If the same reasoning is applied to the law presupposed in the function of consumption, the characteristic  $\phi$ , which prescribes that the marginal propensity to consume is positive and less than 1, is not necessarily found to be true, depending on each individual case. It may so happen that all of the increased portion of income may be consumed in one case, or that more than the increased portion may be spent in another case. In other words it seems that each individual case is governed by a variety of arbitrary factors which may transgress the regularity of a law. Nevertheless, when it comes to the aggregate of many individual cases, i.e. a large majority phenomenon, the existence of the law comes to be clearly accepted, and this can be boiled down to the proposition that *most* of so many cases have the characteristic  $\phi$ . Or similarly statistical generalisation will always arrive at the form showing that "the probability, that an instance taken at random from series  $S$  will have the characteristic  $\phi$ , is  $p$ " (p. 412). Viewed from this standpoint, I should think that there is room to contemplate the problem of the *stability of the propensity to consume*, on which the stability of the economic system was regarded to depend by Keynes, in close relationship with the statistical stability in *A Treatise on Probability*.

Now, if the concept of the consumption function, thinking that aggregate consumption has a stable relationship with aggregate income, is concerned with the existence of a law to be drawn *empirically* from the aggregates, the problem of examining a rational ground for such an

12) J. M. Keynes, *The General Theory of Employment, Interest and Money*, 1936, p. 96.

13) J. M. Keynes, *A Treatise on Probability*, p. 220.

empirical generalisation is still left to be clarified. That it is possible to confirm the stability of the propensity to consume through a statistical procedure is not the answer to the problem; it simply gives its first step. It is because the attempt to clarify in a reasonable and *a priori* way the reason why it becomes so is still left untouched. If stated in a slightly more concrete manner, it is because whether the relationships of regularity drawn from the aggregated group which is indicated in the function of consumption can be consistently connected with the traditional theory of the reasonable preference of consumers or not, in other words whether it is possible to draw the macroscopic theory of the consumption function in a theoretically consistent form from the microscopic theory of consumers' preference or not—such problems, i. e. the so-called aggregation problem in contemporary economics—are still left untouched. In his *General Theory*, Keynes did not refer specifically to this problem, and he did not solve it, either, though it is natural that he could not. However, what is meant by the problem of aggregation, if viewed from a slightly wider angle, is none other than the problem of clarifying the logical ground for the empirical generalisation observable among aggregates. If further put in other words, the problem may also be generally concerned with the logical ground for inductive inference. If it is permissible to view it in this way, for the reason that in the first place the problem of clarifying the logical ground for the inductive inference in general<sup>14</sup>, including the statistical inference, was one of the main issues in Keynes' *Treatise on Probability*, it would be possible as well to think that the methodological foundation provided by Keynes himself with respect to the aggregation problem in contemporary economics had already been perfunctorily given in one sense.

In the meanwhile, needless to say, the problem of aggregation has the nature of being applied not only to the consumption function but also to the so-called aggregate supply function, to the investment demand function also, and even further to the money demand function or liquidity preference function. Now, as the field of view is widened from the market of consumer goods to that of capital goods and further to that of securities, it can be seen that uncertainty comes to intervene in the behaviour of microscopic economic constituents as an overwhelming element which can not be disregarded. Under such circumstances how much appropriateness can the traditional microscopic theory, which generally disregards uncertainty, or which is based on a plain or tacit

14) It seems that Part III, 'Induction and Analogy' in *A Treatise on Probability* and Part V, 'The Foundations of Statistical Inference', particularly Chapter 33, are concerned with this problem.

presupposition that it is possible to predict uncertainty in a numerically measurable form, have as a well-founded basis of the inductive law of regularity confirmed empirically from the majority phenomena that have taken place in the market? In this connection, G. Ackley states that, because the *a priori* analysis in support of Keynes' consumption function (i.e. the theory of the consumers' preference) is no more than "an abstract timeless analysis of rational behavior using a minimum of psychological assumptions", it would be impossible to infer a "generalisation that can be used to predict or describe behavior which is observed over time in a changing world."<sup>15)</sup> Thus, it is interesting that he has expressed his frank doubt as to the *a priori* argument in support of the consumption function. Such doubts, I am sure, will grow stronger and stronger as his eyes are turned from the market of consumer goods which he had in his mind exclusively to the market of capital goods or securities. Although I can not agree with the Ackley's assertion, drawn from his unsophisticated empiricism, which maintains that "the stability.....of the consumption function can only be established empirically"<sup>16)</sup>, as to the very way the aggregation problem is taken up from the aspect of modern economics, I can not help raising a doubt of the same kind as that of Ackley. It seems to me that the only reasonable way of solving the so-called aggregation problem would be to provide the inductive generalisation or inductive inference relating to the behaviour of an aggregate with a justifiable ground *in terms of the theory of probability* by straightforwardly and clearly contemplating the uncertainty in the decision or activity of each individual random member who is a component part of such an aggregate. Be the matter as it may, it appears that the problem of the confrontation of the microscopic system and the macroscopic system is still covered by a thick mist. Viewed from such a way of thinking, since it appears that the systematic thought of Keynes' theory of probability has some meaningful relationship with the above mentioned problem, it would be necessary to make an examination of this point in question by going back to *A Treatise on Probability*.

### III

To begin with I have to give my answer to one question. That is, even if it were a fact that Keynes attached much importance to the concept of probability when he was composing his empirical argument, such a matter is not one that occurred only in the case of Keynes, and

15) G. Ackley, *Macroeconomic Theory*, 1961, p. 220.

16) G. Ackley, *ibid.*, p. 220.

we have still the question, 'Was not the element of uncertainty and the concept of probability similarly considered in fact even in the so-called classical theories?' Is the relationship between Keynes' theory and the theory of probability something fundamentally different in nature, as a matter of fact, in comparison with the relationship between the classical theory and the theory of probability? I shall give my opinion about this question in general here.

I have a firm belief that classical theories had a full acknowledgement of the point that the element of uncertainty did have a more prominent effect on the producers' preference than on the consumers' or more so in the market of capital goods than in the market of consumer goods, and still more so in the field of investment than in other fields. However, it seems to me that classical economics ordinarily completely did away with such uncertainty, and by presupposing the so-called stationary state, or even if they had it in their mind, by presupposing that it was possible to grasp it through either the concept of the numerically measurable probability distributions or probability, they thought that the existence of uncertainty did not call for the necessity of making a fundamental modification of the intrinsic nature of their basic model characterised by its deterministic nature.

In the meantime, because it is too commonly known that many classical economists used to presuppose the stationary state in their arguments, such a point will not be taken up here, but there is, I think, the necessity of making explanation with illustrations about the particular point, in a sense to prevent some misunderstanding too, that they in fact took the above-mentioned uncertainty into account. For example, A. C. Pigou, who succeeded and developed the theories of Marshall, the great master of classical theories, advocated in his *Economics of Welfare* that the uncertainty-bearing on the part of producers should be regarded as one kind of independent factor of production, maintaining that what investment meant in the actual world, where some future event could not be perfectly predicted, was in a sense to expose resources under uncertainty<sup>17)</sup>. Now, the uncertainty that supervenes with a certain type of investment, according to Pigou's view, is to be manifested in its full play in a certain probability distribution of the expected gain of the investment. He calls it "a scheme of prospective returns." Thus, investment can not be anything more than getting possession of a certain probability distribution which is to be manifested in a scheme of prospective returns, by laying out a certain amount of money in a manner

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17) A. C. Pigou, *The Economics of Welfare*, Appendix I, Uncertainty-bearing as a Factor of Production, pp. 771-81.

similar to the case of betting. Now, it was considered that the degrees of probability of each prospective return, making up the component element of such probability distributions, could be clearly made known. Consequently it follows that the 'mathematical expectation' of the probability distributions in question could be interpreted as being something numerically measurable, the mean value of which could be obtainable if the product of each prospective return with the probability of attaining it was added together. However, the prospective return which ought to be a real measure of the uncertainty-bearing should be not the average or actually expected rate of return but that amount by which the latter exceeds the return of a perfectly safe investment (e.g. consols)<sup>18)</sup>. In short, it should not be something vague and non-measurable but a definite and calculable value.

Thus, the uncertainty element in the field of investment, as far as Pigou's view is concerned, is something probable, but because it can after all be grasped in a certain and clear probability distribution, he asserts that its mathematical expectation is calculable. And so it may be a fair guess to say, assuming from his idea of identifying "the most probable return" with "the most frequent return"<sup>19)</sup>, that his thinking is grounded on the theory of statistical frequency<sup>20)</sup> which was integrated by J. Venn. However, even if it was assumed that he took the uncertainty element into account in the field of production or investment, the frame of his deterministic way of thinking with respect to production still remains to be as such without being affected at all. The reason is because, according to Pigou's way of thinking, this particular element of the uncertainty-bearing in the case of investment, is firmly seated in the same analytical structure of marginal productivity in the same capacity and in an exactly similar manner as in the case of three other factors of production, such as nature, labour and capital (i.e. waiting). Therefore, if the above-mentioned view of uncertainty is taken, even if the uncertainty arising out of investment for resources is taken into consideration, it does not necessarily follow, speaking of the problems of the so-called optimum allocation of productive resources, that the nature of the primary model which was composed on the condition of certainty should be fundamentally modified. Needless to say, if Pigou's point of view is taken, the consideration of uncertainty did not come to give any impetus to reorganise the deterministic way of thinking into the way of thinking

18) As to the measure of uncertainty, I decided rather to conform with Robertson's opinion. Cf. D. H. Robertson, *Lectures on Economic Principles*, Vol. 2, p. 101.

19) A. C. Pigou, *ibid.*, p. 774.

20) See *A Treatise on Probability*, pp. 93-100 about J. Venn's view.

in line with the theory of probability.

But Keynes is different. The reason is because the foundation-stone on which his theory was composed is different, however similar the nature of Keynes' way of thinking may superficially appear to be to that of the scholars of the classical school. Though paradoxically it may sound, even if it were proved how far Keynes' way of thinking is different from that of the classical economists, unless such analysis takes up the difference in the foundation on which respective ways of thinking were grounded, it can not be sufficiently complete. Anyway, Keynes never overlooked the specific points that classical theories represented by Pigou, Edgeworth, etc. did take up; not merely the stationary state, which was supposed to be governed by certainty, but the world in which the parts to be played by *change* and *expectation* was sufficiently taken into consideration, i.e. the system in which uncertainty was taken into consideration. However, according to him, in the classical economists' way of thinking, "at any given time facts and expectations were assumed to be given in a definite and calculable form"<sup>21)</sup>, and he further maintained that, even if the uncertainty and risks were recognised, "they were supposed to be capable of an exact actuarial computation."<sup>22)</sup> In other words they thought that by means of "the calculus of probability" uncertainty could be reduced to "the same calculable status as that of certainty itself."<sup>23)</sup> In connection with this Keynes drew a conclusion to the effect that such a way of thinking was of the same type as the methodology which was adopted in the Benthamite philosophy to calculate pleasure and pain, or gain and loss in ethical behaviour.

As far as Keynes' view is concerned, it will lead to the following conclusions that, even if it is supposed that the uncertainty element was taken into consideration in the way of thinking of the classical economists, it was presupposed to have been something definitely calculable through the concept of numerically measurable probability, that the presupposition of mathematical expectation of probability distributions with respect to the so-called reasonable economic decision was no more than the technique of converting uncertainty into certainty, and that through such instrumentality the classical system could not be anything else than a system of certainty in which facts and expectations were grasped in a calculable and definite form at any given time. Thus, all that has been said will lead to the conclusion that the nature of their

21) J. M. Keynes, "The General Theory of Employment", *Quarterly Journal of Economics*, 1937, pp. 212-13.

22) J. M. Keynes, *ibid.*,

23) J. M. Keynes, *ibid.*



system after all has nothing fundamentally different from that type of "system in which the amount of the factors employed was given and the other relevant facts were known more or less for certain,"<sup>24)</sup> i.e. the nature of the model in a stationary state where no uncertainty element is involved at all.

According to Keynes, uncertainty prominent in the field of economics, especially in the field of investment, is a peculiar characteristic inherent not merely in economic phenomena but also in the structure of inference underlying empirical science, which is universally unavoidable in such human behaviour that is being obliged to make alternative judgements of which the results will more or less show up in the future. Especially in the field of economics this characteristic must be substantial as well as decisive. The reason is because among all human behaviour which is influenced by the *preoccupation* with the prospective result, one of the most important ones is something of an economic nature, i.e. "Wealth" or "the accumulation of Wealth." "The whole object of the accumulation of Wealth is to produce results, or potential results at a comparatively distant, and sometimes at an *indefinitely* distant, date. Thus the fact that our knowledge of the future is fluctuating, vague and uncertain, renders Wealth a peculiarly unsuitable subject for the method of classical economic theory."<sup>25)</sup>

It appears that Keynes made use, in a very strict sense, of the term 'uncertainty' which arises in the economic field, particularly in the field of accumulation. It is not something which can be grasped by means of measurable probability as in the case of the classical theory. Since the ratio to win at roulette or in a raffle is calculable probability, as is widely known, uncertainty in this sense is different from that which is meant by Keynes. Viewed from this angle, it is needless to say that the uncertainty which is to be indicated in Pigou's "scheme of prospective returns" does not belong to genuine uncertainty (in the sense meant by Keynes), because in the case of Pigou mathematical expectation is calculable. Uncertainty in the real sense of the word is something neither predictable nor calculable, something to which no measurable probability or even the concept of probability distribution is applicable. Unfortunately, the uncertainty arising in the accumulation of wealth is none other than this kind of uncertainty. Thus, viewed from this angle, it follows that the classical theory is "one of these pretty, polite techniques which tries to deal with the present by abstracting from the fact

24) J. M. Keynes, *ibid.*, p. 212.

25) J. M. Keynes, *ibid.*, p. 213.

that we know very little about the future."<sup>26)</sup> In this connection, because I think it very useful to remember a comment made by D. H. Robertson, successor of A. C. Pigou, in which Robertson frankly expressed his doubts about his master's analysis, it may be cited here as follows: "The ultimate uncertainties of the business life are precisely those which cannot be insured against or pooled in this way. Indeed it is arguable that Pigou's 'scheme of uncertainty' paints too narrow a picture of the whole problem, since often the businessman not only does not know what point on a 'scheme of uncertainty' he will hit, but has only the vaguest idea what the scheme of uncertainty is."<sup>27)</sup>

What is most important for the problem now under discussion is not a theoretically consistent system but how the core of uncertainty in the economic field should be prescribed. To the eyes of Keynes it may have appeared that those classical economic systems which completely disregarded such uncertainty that was beyond the probability-calculation, must have been no more than "market-place idols", in the sense that they did not penetrate to the core of substantial uncertainty inherent in economic phenomena, however theoretically consistent they might have been in themselves. However, it must be admitted that if our eyes are cast upon aggregated economic activities, something of regularity can be empirically confirmed from a macroscopic point of view, through the mechanism of the market. However, if our eyes are once turned from the microscopic view to the foundation of the preferences of individuals, we are confronted with an entity of uncertainty as if it were a mollusc which we can not take hold of. Thus, we are again confronted with the conception of uncertainty which has its root deep in the background of the problem arising out of the opposition between the microscopic and macroscopic ways of thinking<sup>28)</sup>.

Now, it seems to me that the difference in the ways of thinking with respect to the concept of uncertainty and probability held by Keynes and the classical economists can be concretely and straightforwardly seen in the theory of money and the rate of interest. In other words it is said that this very difference is in itself an outstanding

26) J. M. Keynes, *ibid.*, p. 215.

27) D. H. Robertson, *op. cit.*, p. 105.

28) About such a point at issue there seems to be a great deal to be learned from the logic of modern physics, particularly statistical dynamics and the so-called principle of uncertainty. But it is needless to say that there is a necessity of making a clear distinction between uncertainty in natural phenomena and in human phenomena. (See Kotaro Toyama, *Logic of Today's Physics (Gendai Butsurigaku no Ronri)*, 1956, Chapter 3, 'Logic of Statistical Dynamics'; Shinichiro Tomonaga, *Quantum Dynamics (Ryoshi Rikigaku)*, Chapter 9, 'State of Quantum Dynamics', particularly pp. 296-31). The writer owes the reference materials and information about the essential problems in today's physics to the advice of Mr. Kajita of the Department of Economics, Shiga University.

characteristic of the Keynesian theory of inducement to invest or the theory of investment decisions, in which the above-mentioned theory is included as one train of thought. But, before taking up such a proposition now, I shall turn to the way of thinking in *A Treatise on Probability* and see how Keynes generally grasped the concept of uncertainty and probability.

#### IV

As far as I can see, it appears that the concept of uncertainty or probability of Keynes' own creation is expressed in a very vivid manner in his criticism of "the theory of statistical frequency" which was advanced in his *Treatise on Probability*<sup>29)</sup>. Taking this as a clue, I shall now make my approach to this problem. Before going into any further discussion, let us not forget that whenever the classical economists took uncertainty into account, they grounded their reasoning on the concept of objective probability on the presupposition of the theory of statistical frequency and the resulting concept of uncertainty which was prescribed by it.

Now, the essence of the theory of statistical frequency can be briefly put as follows: Taking the uncertainty-bearing of Pigou already referred to as an example, the proposition that the probability of producing a prospective return of ¥ 10 by spending ¥ 100 for a certain investment is  $f/n$ , means that this particular investment is one of a group of a great number of investments of the same type as the one mentioned—the  $f/n$  ratio of which produces the return of ¥ 10. And it follows that the very existence of a series of investments having the frequency of  $f/n$  to produce a prospective return of ¥ 10 is purely an empirical phenomenon, and they should be determined exactly in the same way as in the case of any other matter of fact. Viewed from this angle, probability is something ultimately concerned with a series of certain events in the form of frequency in respect of a certain characteristic. Then, the problems relating to probability come to be something to indicate intensively the results of experience (for instance what has been experienced about the aggregate of a certain type of investment)<sup>30)</sup>.

If based on such theory, probability must be understood to be characterised by an objective frequency which is inherent in the aggregate of

29) Criticism of the frequency theory developed in *A Treatise on Probability* (pp. 92-110) is, if definitely spoken, directed against the view of Venn's *Logic of Chance* (1st ed., 1866) and 'the generalised frequency theory'. It can be assumed that the latter is principally composed of the views held by Whitehead (which was given to Keynes in the form of manuscripts).

30) Cf. J. M. Keynes, *A Treatise on Probability*, pp. 94-95.

all the cases experienced, and it follows that it should have some definitely known numerical values in all cases rather than that it might have. Now, when it is said in our daily argument that there is no knowing about probability or that probability is vague, it can fairly be said that it is all because of our poor or incomplete knowledge of probability and not because of objective probability itself. For example, the reason why the probability of a certain type of investment to produce certain prospective returns is not clear is not to be ascribed to the characteristics of probability distribution itself of the said investment-returns, but is to be ascribed to the incompleteness of our knowledge or information with respect to the matter. The reason is, if based on the frequency theory, because "the probabilities are relative, not to our knowledge, but to some objective class, possessing a perfectly definite truth-frequency" (p. 102). In other words probability is something provided objectively to the events which have been experienced. In a word it means *objective* probability.

What is meant by probability by Keynes differs substantially from this. Probability, according to him, is the probability of an inference or an argument. It is the probability not of events but of propositions<sup>31)</sup>. As in the case of the frequency theory, probability is not something objectively innate to the aggregate of events. It is "in an important sense *relative to given premisses*" (p. 102) as the probability of the conclusion of a certain inference. Putting it another way, it is concerned with actual or hypothetical information or knowledge which forms the presupposition of inference. Now, in just as much proportion to the difference among informations or knowledges on which the presupposition of inference is to be formed, the extent of the probability of that proposition may vary. In short, Keynes' probability is a subjective probability. In this sense it is also something concerned with "the degree of rational belief" in a proposition. But, it is not something based on inference in order to draw a conclusion with "demonstrative certainty" as in the case of the geometry of ideal space or a syllogism, though it can be called rational belief. It is something based on the "arguments from premisses leading to conclusions which are reasonable but not certain," (pp. 99, 217), i. e. a probable and reasonable inference. Keynes further

31) In an argument or inference, supposing that its presumption is made up of any set of propositions  $h$ , that its conclusion is made up of another set of propositions  $a$ , and that the degree of rational belief of the conclusion reached from such presumption  $h$  is  $\alpha$ , then it can be expressed as  $a/h = \alpha$ . This formula indicates the probability of conclusion  $a$  drawn from the presumption  $h$ . Thus, probability as conceived by Keynes is the expression of rational belief in relation to a certain proposition (i.e.  $a$ ) and it is necessarily based on the proposition which comprises the presupposition ( $h$ ).

states that almost all of the empirical sciences and our daily behaviour depend on such a probable inference. In this way it is seen that Keynes' theory of probability is not something concerned with the nature inherent in the objective *events* in themselves, as in the case of the frequency theory, but with the nature of subjective *inference* with respect to the events. In other words it treats the varied degrees of confidence aroused from the inference of drawing an uncertain, though reasonable, conclusion.

Viewed from such a standpoint, what defects could there be in the theory of statistical frequency? The concept of probability held in the frequency theory is clearly understandable by anybody because of its objective and measurable nature, but it can not throw light on the process of forming our reasonable belief itself. In other words, it can not be really useful in making our decision. Now, in the ordinary activities of daily life and also in almost all of the empirical sciences, however differently they may be characterised in their abstractness, universality or consistency, the process of inference based on experience lies deeply rooted at the bottom. In short, our activities or sciences are closely connected with the process of empirical inference. Moreover, the meaning of such inference can not be after all anything else other than the preference for a certain conclusion to be drawn through a reasonable procedure from a given presupposition. Thus, fundamentally speaking, reasonable *inference* means reasonable *preference*, which can be seen in the statement made by Keynes to the effect that "probability is the study of the grounds which lead us to entertain a *rational* preference for one belief over another" (p. 97). It is needless to say that many of the reasonable inferences used in our daily life or empirical sciences aim at no demonstrative certainty, but do have a more or less probable nature, because there can not be any room to make a preference to 'alternatives' in the process of drawing a definite conclusion by demonstration, as in the case of geometry.

In the meanwhile, if the basic nature of probable inference lies in determining "which alternatives it is *reasonable to prefer*" (p. 98), then probability comes to be something concerned with the "logical grounds of decision" and it leads us to the conclusion that, if such a reasonable decision were concealed in the core of our life, then probability would be something that comes in contact with the very roots of both ordinary life and scientific life. If viewed from such an angle, statistical probability can not be useful either as "the guide of life" or as "the guide of decision" (pp. 6, 96). Moreover, "in following it we are not acting according to reason" (p. 96). In short, it comes down to the fact that statistical probability can not be a sufficient condition for *reasonable pre-*

*ference*. This can well be guessed to be the meaning of Keynes' statement to the effect that "We may call a statistical frequency a probability, if we choose; but the fundamental problem of determining which of several alternatives is logically preferable still awaits solution" (p. 98).

Nevertheless, what is the reason for saying that statistical probability is not sufficient to be made the logical ground of our preference? I shall give a further explanation about this point by taking the analysis made by Pigou as an example again. Illustrations with respect to *a certain type* of investment which takes *a certain type* of probability distribution of prospective returns are given in Pigou's analysis. However, there is a risk in such a simplification of missing the very core of the decision for investment, because what is important for anybody who makes the decision to invest is the selection among several types of investment. Now then, let's suppose that there are so many types of investment and each of them has a probability distributions of its own type respectively. Thus, what is important for the decision of investors is a selection among so many mathematical expectations which are substantiated by their own inherent statistical frequency. Then, it follows that each objective probability distribution (of prospective returns) in correspondence to the difference due to the type of investment is taken for granted in this case. In other words, it is not put in question how the statistical frequency (objective probability), on which our preference or decision is presupposed, comes to be formed. But, when the process through which the above-mentioned probability distribution is formed is further analysed, it can easily be understood that a number of factors affect the foundation of the 'real value' of each statistical probability, which may govern our decision in some measure. Therefore, if viewed from subjective probability, the probability of statistical frequency peculiar to each type of investment should be put in question. In other words the *probability* (in the sense held by Keynes) of the proposition, saying that the probability of producing a given prospective return through a given type of investment is  $p$ , should be put in question. Be the matter as it may, it is sure enough that the order of the mathematical expectation peculiar to each type of investment does not precisely reflect that of the preference for the decision to make an investment. It seems to me, as referred to before, that Keynes cherished the opinion that the classical economists' thoughts were based on Benthamite logic. What is meant by such a comment is that even in Benthamism it was presupposed that the order of preference for an ethical decision was supposed to correspond precisely to that of the mathematical expectations (of the expected virtue or pleasure), and that the behaviour-pattern of this type was completely

taken over as the foundation of an economic decision by the classical economists. Consequently it might be said that if this viewpoint is taken, the behaviour-pattern presupposed by Pigou in the background of the investment decision is substantially grounded on Benthamite logic<sup>32</sup>.

Be the matter as it may, because the mathematical expectations substantiated by statistical probability can not be a sufficient condition for forming a logical foundation for our decision, there arises a necessity of building up a new systematised way of thinking of probability, in which all the factors that have been given away for the reason of being subjective or non-measurable from the aspect of the theory of statistical frequency should be comprehended. The reason is because such a system of thought can only be really useful as a 'logical ground of decision' or as a 'guide of life'. Keynes' statement to the effect that "My Treatise is concerned with the *general* theory of arguments from premisses leading to conclusions which are reasonable but not certain" (p. 98) can be interpreted in such a way (*italic mine*).

In the meantime, probability in the theory of statistical frequency is characterised by its clearness and measurableness. Perhaps it is because of such characteristics that this theory has come to be generally accepted. But what really matters lies neither in a clear definition of probability nor in the logical consistency of a way of thinking, but *in finding a logical ground for our decision or preference* having its root deep in the bottom of our inference. If viewed from such a standpoint, even such a problem as to whether the probability of a conclusion of our inference is measurable or not is not a substantial matter. However, in many instances probability which is concerned with the degree of our rational belief is not measurable. Frequently it is something which can not even be compared. In a case like this "Nor have we any *prima facie* indications of the existence of a common unit to which the magnitudes of all probabilities are naturally referable. A degree of probability is not composed of some homogeneous material, and is not apparently divisible into parts of like character with one another" (p. 30).

It is, needless to say, with his criticism against the fundamental conception of the classical economists, on the one hand, that Keynes limited the meaning of probability very strictly, finding it impossible to grasp the real uncertainty in the economic field through the concept of a

32) The criticism of Benthamite logic comprises the major subject of Chapter 26 (The Application of Probability to Conduct, pp. 307-23) of *A Treatise on Probability*. Keynes came to acquire his unique view holding that the conduct-pattern in Benthamism is an application of the classical probability theory from *Principia Ethica* written by G. E. Moore, with whom Keynes was well acquainted. The criticism of Benthamism developed in this chapter seems to have formed the original idea of his later criticism of classical economic thought.

measurable and objective probability, but we should not overlook on the other hand that such an idea sprang from a motif in *A Treatise on Probability*, which maintained that the concepts of uncertainty and probability which lie deep in the logical ground for decision or preference were neither measurable nor comparable. Leaving the matter as it is, it appears to me that Keynes had an idea that the concepts of uncertainty or probability in the economic field make numerical comparison impossible in the substantial meaning of the word. If viewed in this way, Keynes' sharp criticism in his *Treatise on Probability* of the theory of statistical frequency, the motive of which was to discover the 'logical ground for decision', was the first criticism ever made of classical economic thought which took up the problem of uncertainty and probability in this theory, and at the same time it was consistent in its groundwork with his later criticism of classical thought<sup>33</sup>.

Another criticism raised by Keynes against the theory of statistical probability is his opinion that the concept of objective probability can not be a reasonable ground for empirical inductive inference. As already mentioned before, to seek 'the logical justification' of inductive inference in general, including the statistical inference in the concept of probability, i.e. to attempt to reinterpret the 'inductive law' in terms of the theory of probability, was part of his main subject matter. What is regarded as important for this subject matter lies not in whether the way of thinking in the theory of statistical frequency has logical conformity or not, but whether "the body of probable argument, upon which the greater part of our generally accepted knowledge seems to rest can be explained in terms of it (the frequency theory)" (p. 109). Can it be possible, as a matter of fact, that such an inference can be drawn from the assumption of statistical frequency? Keynes gave a negative answer with respect to this question after making an examination, for example, of various pre-suppositions on which the conclusion of Darwin's *The Origin of Species* was based, which may be quoted as follows: "Not only in the main argument (in 'The Origin of Species'), but in many of the subsidiary discussions, an elaborate combination of induction and analogy is superimposed

33) Harrod, by asserting in a note contributed to *The Life of J. M. Keynes* in respect to the *Treatise on Probability* that "in my own person I cannot resist some uneasiness in regard to the indefinability of probability, and hanker after some form of the Frequency theory" (R. F. Harrod, *ibid.*, p. 653) does not agree with Keynes' concept of subjective probability. In all likelihood it appears to me that the aforementioned point has something to do with the complete neglect of the problem of uncertainty meant by Keynes when Harrod presented a deterministic model in his attempt to establish a dynamic economics. As to the point that all post-Keynesians have in general deviated from the problem of uncertainty arising out of economic decision, with the exception of J. Robinson, it can be added that the support of Professor Schackle can be much hoped for. Schackle, *op. cit.*, p. 29.



upon a narrow and limited knowledge of statistical frequency. And this is equally the case in almost all everyday arguments of any degree of complexity. The class of judgements, which a theory of statistical frequency can comprehend, is too narrow to justify its claim to present a complete theory of probability." (p. 109)

Thus, it leads us to the conclusion that the problem of rebuilding the logical ground for inductive inference through a new concept of subjective probability, instead of statistical objective probability, was still left to be solved by Keynes. What forms the main subject of Part III of *A Treatise on Probability* is none other than this problem. Through all that has been discussed I have shown the general nature of the concept of uncertainty and probability held by Keynes, in comparison with that of objective probability, which had its root deep in the foundation of classical thought. Then, it is self-explanatory from the composition of the problem briefly stated at the beginning of Section II that how the theory of inducement to invest, or the theory of investment decisions, maintained by Keynes should be recomposed based on the above mentioned new concept of subjective probability is our next problem to be solved.